Class 9th Mathematics
Module 1/1

Introduction to Euclid’s Geometry

by

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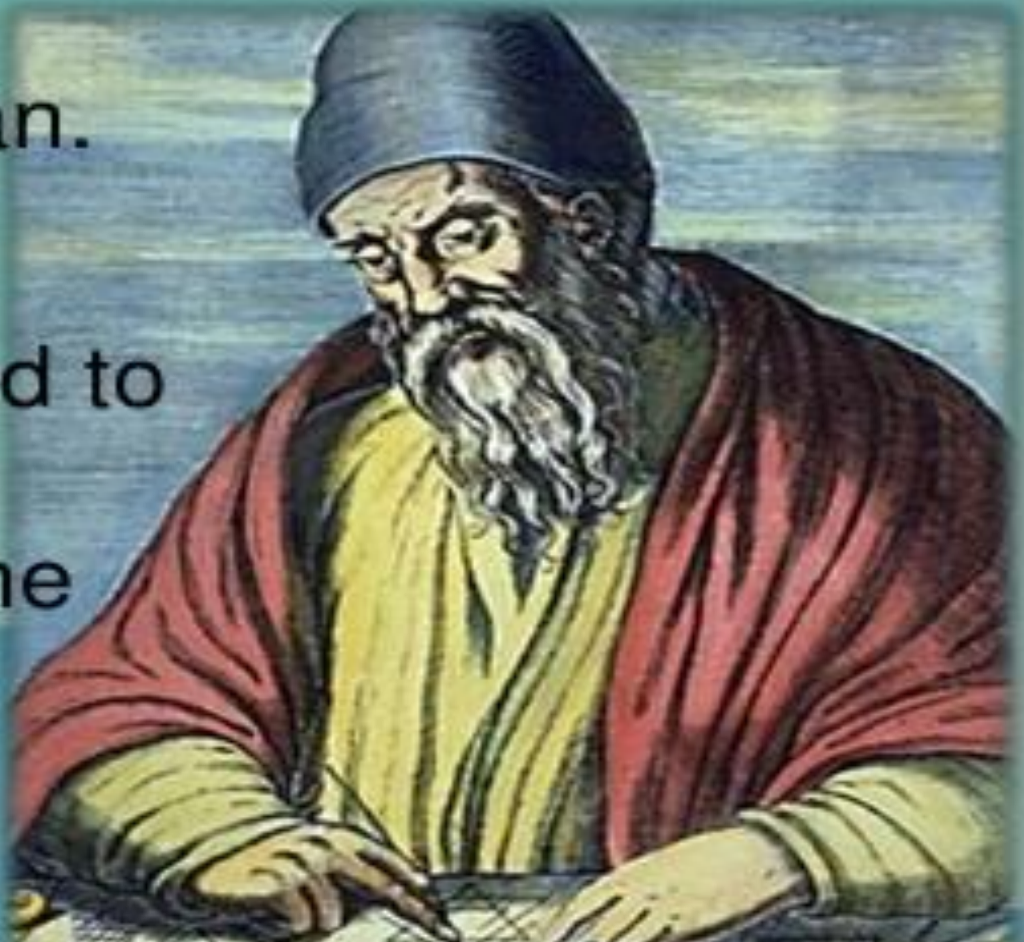
ABOUT EUCLID

- Euclid, sometimes called Euclid of Alexandria to distinguish him from Euclid of Megara
- He was a Greek Mathematician.
- Euclidean Geometry is a mathematical system attributed to the Alexandrian Greek Mathematician Euclid, which he described in his textbook on geometry: The Elements.



See the Bold *Shaddow of* Urania's *Glory*,
Immortall *in His* Race, *no lesse in* Story:
An Artist *without* Error; *from whose* Lyne,
Both Earth and Heav'ns, *in sweet* Proportions twine:
Behold Great EUCLID: But behold him well!
For 'tis in Him Divinity doth dwell.

G. Wharton.

EUCLID'S ELEMENTS OF Geometry.

In XV. Books:
With a supplement of divers *PROPOSITIONS* and *COROLLARIES*.
To which is added, a *Treatise* of REGULAR SOLIDS.
By *CAMPANE* and *FLUSSAS*.
LIKEWISE

Euclid's DATA.

And MARINUS his Preface thereunto annexed.

Also a Treatise *of the* Divisions *of* Superficies, *ascribed to* Machomet Bagdedine, *but published by* Commandine, *at the request of* John Dee *of* London; *whose* Preface *to the said* Treatise *declares it to be the* Worke *of* EUCLIDE, *the* Author *of these* ELEMENTS.

Published by the Care and Industry of
JOHN LEEKE and GEORGE SERLE, Students in the *MATHEMATICKS*.

LONDON:
Printed by R. & W. LEYBOURN, for GEORGE SAWBRIDGE at the *Bible* upon *Ludgate-hill*,
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EUCLID'S GEOMETRY

GEOMETRY

GEO — Earth

METREIN — To measure

- Geometry originated in Egypt as an art of Earth measurement
- **Euclid** (325 BCE-265 BCE): The Father of Geometry
- The first **Egyptian mathematician** who initiated a new way of thinking the study of geometry.
  Introduced the method of proving a geometrical result by **deductive reasoning** based upon previously proved result & some self evident specific assumptions called axioms.

- Euclidean Geometry is the study of Geometry based on definitions, undefined terms,(points, line and plane) and the assumptions of the mathematician Euclid(330BC)
- Euclidean geometry is the study of flat space.
- Euclidean text Elements was the first systematic discussion of geometry. While many of Euclid's findings had been previously stated by earlier Greek mathematicians, Euclid is credited with the developing of the first comprehensive deductive system. Euclid`s approach to geometry consisted of proving all the theorems from a finite number of postulates and axioms.
- The concept in Euclid`s geometry remained unchallenged until the early 19th century. At that time , other forms of geometry started to emerge, called non-Euclidean geometry. It was no longer assumed that Euclid's geometry could be used to describe all physical space.

Basic Concepts and Important Points

- **Postulates:** The basic facts which are taken, for granted, without proof and which are specific to geometry are called postulates.
- **Axioms:** The basic facts which are taken for granted, without proof and which are used throughout in the mathematics are called axioms.
- **Theorem:** The conclusions obtained through logical reasoning based on previously proven results and some axioms constitute a statement known as a theorem or a proposition.
- **Point:** A point is represented by a fine dot made by a sharp pencil on a sheet of paper.

- **Plane:** The surface of a smooth wall or the surface of a sheet of paper or the surface of a smooth black board are close examples of a plane.
- **Line:** A line is breathless length e.g.. if we fold a piece of paper, the crease in the paper represents a geometrical straight line. The edge of a ruler, the edge of the top of a table, the meeting place of two walls of a room are some examples of a geometrical straight line.
- **Incidence Axioms:**

Axiom 1: A line contains infinitely many points.

Axiom 2: Through a given point, infinitely many lines can pass through.

Axiom 3: In given two points A and B, there is one and only one line that contains both the points.

- **Collinear Points:** Three or more points are said to be collinear, if there is a line which contains all of them.
- **Concurrent Lines:** Three or more lines are said to be concurrent, if there is a point which lies on all of them.
- **Intersecting Lines:** Two lines which meet at one point are said to be intersecting lines. The common point is called the ‘point of intersection’.

Note: Two distinct lines cannot have more than one point in common.

- **Parallel Lines:** Two lines l and m in a plane are said to be parallel lines, if $l \cap m = \phi$. If l and m are two parallel lines in a plane, we can write $l \parallel m$.

- Parallel Axiom: If l is a line and P is a point not on line l, there is one and only one line m which passes through P and is parallel to l.

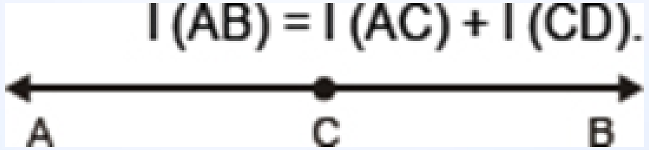
- Two lines which are both parallel to the same line, are parallel to each other.
- If l, m, n are three lines in the same plane such that l intersects m and n ll m, then l intersects n also.
- If l, m, n are three lines in the same plane such that l intersects m and n ll m, then l intersects n also.
- If land m are intersecting lines, l ll p and q ll m, then p and q also intersect.
- If lines AB, AC, AD and AE are parallel to a line l, then points A, B, C, D and E are collinear.

- Line Segment: In given two points A and B on a line l, the connected part (segment) of the line with end points at A and B, is called the line segment AB.
- Interior Point of a Line Segment: A point P is called an interior point of a line segment AB, if P ε AB but P is neither A nor B.
- Congruence of Line Segments: Two line segments AB and CD are congruent, if the trace-copy of one can be superposed on the other so as to cover it completely and exactly.
- Line Segment Length Axiom: Every line segment has a length. It is measured in terms of 'metre' or its parts.

- Congruent Line Segment Length Axiom: Two congruent line segments have equal length and conversely, two line segments of equal length are congruent,

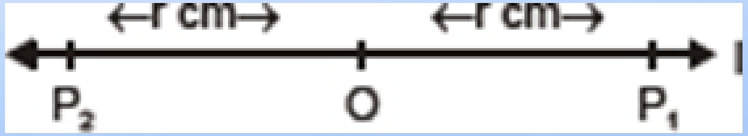
  i.e., $AB \cong CD \Leftrightarrow l\,(AB) = l\,(CD)$.

- Line Segment Addition Axiom: If C is any interior point of a line segment AB, then



- Line Segment Construction Axiom: Given a point O on a line l and a positive real number r, there are exactly two points P_1 and P_2 on l, on either side of O such that

  $l\,(OP_1) = l\,(OP_2) = r$ cm.



- Distance between Two Points: The distance between two points P and Q is the length of the line segment joining them and it is denoted by PQ.
- Betweenness: Point C is said to lie between the two points A and B, if, (a) A, B and C are collinear points and
  (b) AC + CB = AB.

- Mid-point of a Line Segment: Given a line segment AB, a point M is said to be the mid-point of AB, if M is an interior point of AB such that AM = MB.

Line through M, other than line AB is called the bisector of the segment AB.

- Opposite Rays: Two rays AB and AC are said to be opposite rays if they are collinear and point A is the only common point of these two rays.

Note: Two rays or two line segments or a line segment and a ray (line) are said to be parallel, if the lines containing them are parallel.

- Euclid's Five Postulates:

(a) A straight line may be drawn from any one point to any other point.

(b) A terminated line can be produced indefinitely.

(c) A circle can be drawn with any centre and any radius.

(d) All right angles are equal to one another.

(e) If a straight line falling on two straight lines makes the interior angles on the same side of it, taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.

- Some Euclid’s axioms:
  - (a) Things which are equal to the same thing are equal to one another.
  - (b) If equals are added to equals, the wholes are equal.
  - (c) If equals are subtracted from equals, the remainders are equal.
  - (d) Things which coincide with one another are equal to one another.
  - (e) The whole is greater than the part.
  - (f) Things which are double of the same things are equal to one another.
  - (g) Things which are halves of the same things are equal to one another.
- A system of axioms is called consistent, if it is impossible to deduce from these axioms a statement that contradicts any axioms or previously proved statement.

Some worked out examples:-

1. Which of the following statements are true and which are false?
Give reasons for your answers.

(i) Only one line can pass through a single point.
(ii) There are an infinite number of lines which pass through two distinct points.
(iii) A terminated line can be produced indefinitely on both the sides.
(iv) If two circles are equal, then their radii are equal.
(v) if AB = PQ and PQ = XY, then AB = XY.

Solution: (i) False

There can be infinite number of lines that can be drawn through a single point. Hence, the statement mentioned is False

(ii) False

Through two distinct points there can be only one line that can be drawn. Hence, the statement mentioned is False

(iii) True

A line that is terminated can be indefinitely produced on both sides as a line can be extended on both its sides infinitely.

Hence, the statement mentioned is true.

(iv) True

The radii of two circles are equal when the two circles are equal. The circumference and the centre of both the circles coincide; and thus, the radius of the two circles should be equal.   Hence, the statement mentioned is True.

(v) True

According to Euclid's 1st axiom- "Things which are equal to the same thing are also equal to one another".

Hence, the statement mentioned is true.

2. Give a definition for each of the following terms.
Are there other terms that need to be defined first? What are they, and how might you define them?
(i) parallel lines (ii) perpendicular lines
(iii) line segment (iv) radius of a circle (v) square

Solution:

Yes, there are other terms which need to be defined first, they are:

Plane: Flat surfaces in which geometric figures can be drawn are known are plane. A plane surface is a surface which lies evenly with the straight lines on itself.

Point: A dimensionless dot which is drawn on a plane surface is known as point. A point is that which has no part.

Line: A collection of points that has only length and no breadth is known as a line. And it can be extended on both directions.

A line is breadth-less length.

(i) Parallel lines- Parallel lines are those lines which never intersect each other and are always at a constant distance perpendicular to each other. Parallel lines can be two or more lines.

(ii) Perpendicular lines- Perpendicular lines are those lines which intersect each other in a plane at right angles then the lines are said to be perpendicular to each other.

(iii) Line Segment- When a line cannot be extended any further because of its two end points then the line is known as a line segment. A line segment has 2 end points.

(iv) Radius of circle- A radius of a circle is the line from any point on the circumference of the circle to the centre of the circle.
(v) Square- A quadrilateral in which all the four sides are said to be equal and each of its internal angle is right angles is called **SQUARE.**

3. Consider two ‘postulates’ given below:
(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.
(ii) There exist at least three points that are not on the same line.
Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid’s postulates? Explain.

Solution:

Yes, these postulates contain undefined terms. Undefined terms in the postulates are:

There are many points that lie in a plane. But, in the postulates given here, the position of the point C is not given, as of whether it lies on the line segment joining AB or not.

On top of that, there is no information about whether the points are in same plane or not. Yes, these postulates are consistent when we deal with these two situation: Point C is lying on the line segment AB in between A and B.

Point C does not lie on the line segment AB. No, they don't follow from Euclid’s postulates. They follow the axioms.

4. If a point C lies between two points A and B such that AC = BC, then prove that AC = $\mathbf{1}/\ \mathbf{2}$ AB.

To prove:

If a point C lies between two points A and B such that AC = BC, then prove that AC = $\mathbf{1/2}$ AB.

Solution :-

Given that, AC = BC

Now, adding AC both sides.

L.H.S + AC = R.H.S + AC

AC + AC = BC + AC

2AC = BC + AC

We know that, BC +AC = AB (as it coincides with line segment AB)

∴ 2 AC = AB (If equals are added to equals, the wholes are equal.)

⇒ AC = $\mathbf{1}\,/\mathbf{2}$ AB.

5. Why is Axiom 5, in the list of Euclid’s axioms, considered a ‘universal truth’?
(Note that the question is not about the fifth postulate.)

Solution:

Axiom 5 : The whole is always greater than the part. For Example:, A cake. When it is whole or complete, assume that it measures 2 pounds but when a part from it is taken out and measured, its weigh will be smaller than the previous measurement. So, the fifth axiom of Euclid is true for all the materials in the universe. Hence, Axiom 5, in the list of Euclid’s axioms, is considered a ‘universal truth’.

6. How would you rewrite Euclid’s fifth postulate so that it would be easier to understand?

Solution:

Euclid’s fifth postulate:

If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles. i.e., the Euclid’s fifth postulate is about parallel lines. Parallel lines are the lines which do not intersect each other ever and are always at a constant perpendicular distance apart from each other. Parallel lines can be two or more lines. A: If X does not lie on the line A then we can draw a line through X which will be parallel to that of the line A. B: There can be only one line that can be drawn through the point X which is parallel to the line A.

Thank you

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